

Chiral phase transition of bulk Abelian gauge theories in the Randall-Sundrum brane world

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Abstract

The chiral phase transition of strong-coupling Abelian gauge theories is investigated in the brane world. It is assumed that gauge boson propagates in an extra dimension, i.e. bulk gauge theories. The phase structure is analytically evaluated by using the low-energy effective theories. We also numerically solve the ladder Schwinger-Dyson equation for the full fermion propagator including Kaluza-Klein (KK) excitation modes of the gauge field. It is found that the chiral phase transition is of the second order, and the critical value of the coupling constant is obtained. The extra dimension has a large influence on the chiral phase transition for the Randall-Sundrum (RS) brane world. It is studied how the number of KK modes affect the chiral phase transition.

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I. INTRODUCTION

The idea of compact extra dimensions has been proposed to solve the hierarchy problem between the Planck scale and the electroweak scale in Ref. [1–3]. To avoid fine tuning the radius of compact extra dimensions, Randall and Sundrum introduced Minkowski branes in five-dimensional anti-deSitter space-time which is a static solution to Einstein’s equation [4]. At the beginning, it has usually been assumed that the standard model particles are confined in the Minkowski brane while the graviton can propagate in extra dimensions. In the present paper we study the influence of bulk gauge field on chiral phase transition.

A possible mechanism to confine the standard model particles in the brane has been found by constructing a bulk standard model [5]. In the bulk standard model KK excitation modes [6] of the standard model particles appear in the brane. A variety of low energy phenomena have been studied in the bulk standard model. One of the most interesting phenomena is found in spontaneous electroweak symmetry breaking of gauge theories. It may play a decisive role in the standard model and the grand unified theories. The dynamical mechanism of electroweak symmetry breaking has been reexamined within brane world models [7–12]. It has been pointed out that the exchange of KK modes enhances dynamical symmetry breaking. A possibility of the top quark condensation is revived along with the brane world scenario [13,14]. Dynamical symmetry breaking has been evaluated in higher-dimensional compact space-time, and then the result is reduced to the four-dimensional branes. But the role of the brane particle in symmetry breaking is not clear in previous studies. The brane particle may have a non-negligible effect on symmetry breaking [15].

There are a variety of models for the brane world. In many models it is assumed that compact extra dimensions exist. It has been shown that the finite size effect and the boundary condition can dramatically change the phase structure of dynamical symmetry breaking [16,17]. The RS brane world has two Minkowski branes in five-dimensional negative curvature space-time, AdS^5 . The space-time curvature also has an important effect on the phase structure of dynamical symmetry breaking [18,19]. Especially in negative curvature space-time it has been shown that chiral symmetry is always broken down for models with four-fermion interactions if the space-time dimensions are less than four [16,18,20]. Therefore, we launched a plan to study dynamical symmetry breaking in the RS brane world using the induced four-dimensional Lagrangian from five-dimensional gauge theory. In Refs. [10] and [12] the structure of chiral symmetry breaking has been studied in the bulk fermion models. It is pointed out that the dynamical fermion mass is much smaller than the Planck scale.

In this paper we consider five-dimensional Abelian gauge theories as a simple model where chiral symmetry is broken down dynamically. The structure of chiral phase transition is investigated in the bulk gauge theories. In §2 we assume that gauge boson propagates in an extra dimension, i.e. bulk gauge theories. The explicit expressions of the induced four-dimensional Lagrangian are obtained. In §3 we construct low-energy effective theories with local four-fermion interactions. Using the bifurcation method phase structure is analytically investigated in the effective theories. In §4 we numerically solve the ladder Schwinger-Dyson equation including KK modes of the gauge field.

II. INDUCED FOUR-DIMENSIONAL LAGRANGIAN OF BULK GAUGE THEORIES

Throughout this paper we assume an existence of five-dimensional bulk gauge fields, while the fermion fields are confined in the brane. We consider the Abelian gauge theories with massless fermions. Since we are interested in KK excitations for gauge fields, the KK excitations for graviton are assumed to have no serious effect on chiral symmetry breaking, and thus are ignored. We take account of KK excitations for gauge fields only, and construct the induced four-dimensional Lagrangian from the five-dimensional theory in the brane.

If the gauge fields are confined in the brane, an extra dimension does not contribute to gauge interactions. The Lagrangian for Abelian gauge theories has a familiar form:

$$\mathcal{L}^{(4)} = \frac{1}{2} A_\mu \left[\eta^{\mu\nu} \partial^2 - (1 - \xi) \partial^\mu \partial^\nu \right] A_\nu + \bar{\psi} (i \partial_\mu + e A_\mu) \gamma^\mu \psi. \quad (2.1)$$

It has been known for a long time that a fermion acquires the dynamical mass and that chiral symmetry is broken down for strong coupling, $\alpha = e^2/(4\pi) > \pi/3$ [21,22].

Below, we consider the bulk gauge field which propagates in an extra dimension. We then construct the induced four-dimensional Lagrangian.

A. Bulk Abelian gauge theories in flat extra dimension

We first study a flat space-time with a single orbifold extra dimension, $M^4 \otimes S^1/Z_2$. The extra dimension is assumed to be compactified as a circle with radius R . According to the S^1/Z_2 invariance, we adopt the following boundary conditions,

$$A_\mu(x, y+1) = A_\mu(x, y), A_\mu(x, y) = A_\mu(x, -y), \quad (2.2)$$

$$A_4(x, y+1) = A_4(x, y), A_4(x, y) = -A_4(x, -y), \quad (2.3)$$

where x is a coordinate on four-dimensional Minkowski surfaces, y is a coordinate in an extra dimension normalized by $2\pi R$ and $\mu = 0, 1, 2, 3$. In the branes, $y = y^* (= 0)$, so these conditions simplify

$$\partial_4 A_\mu(x, y = y^*) = 0, \quad (2.4)$$

$$A_4(x, y = y^*) = 0. \quad (2.5)$$

The Z_2 symmetry prevents the gauge field A_4 from having non-vanishing value in the brane. The field A_4 cannot couple with the brane particle directly. And furthermore, we apply the gauge fixing conditions where the extra component of the gauge fields, A_4 , is decoupled from A_μ . (See Appendix A.) Thus, the fifth component of the gauge fields disappears from the Lagrangian on the brane [24,25].

By adopting the periodic boundary condition and the Z_2 invariance, the bulk gauge field is decomposed into Fourier modes:

$$A_\mu(x, y) = \frac{1}{\sqrt{2\pi R}} \sum_n A_\mu^{(n)}(x) \cos(2\pi n y). \quad (2.6)$$

The induced four-dimensional Lagrangian is described by

$$\begin{aligned}
\mathcal{L}^{(4)} &= \int_0^{1/2} dy \left[-\frac{1}{4} F^{MN} F_{MN} + \mathcal{L}_{\text{G.F.}} + \mathcal{L}_{\text{F}} \delta(y - y^*) \right] \\
&= \frac{1}{2} \sum_n A_\mu^{(n)} \left\{ \eta^{\mu\nu} \left[\partial^2 + \left(\frac{n}{R} \right)^2 \right] - (1 - \xi) \partial^\mu \partial^\nu \right\} A_\nu^{(n)} \\
&\quad + \bar{\psi} \left(i \partial_\mu + e \sum_n A_\mu^{(n)} \right) \gamma^\mu \psi,
\end{aligned} \tag{2.7}$$

where $\mathcal{L}_{\text{G.F.}}$ represents the gauge fixing term and \mathcal{L}_{F} is the Lagrangian for massless fermion. The contribution from the extra dimension is imposed in the KK modes, $A_\mu^{(n)}(x)$, with mass n/R .

B. Bulk Abelian gauge theories in warped extra dimension

The existence of the brane naturally leads to the curved extra dimensions. L. Randall and R. Sundrum found the curved space-time with Minkowski branes as a static solution of the Einstein equation [4]. The model has *two* Minkowski branes at the S^1/Z_2 orbifold fixed points, $y^* = 0, 1/2$, and the AdS_5 between these branes:

$$ds^2 \equiv G_{MN} dx^M dx^N = e^{-2kb_0|y|} \eta_{\mu\nu} dx^\mu dx^\nu - b_0^2 dy^2, \tag{2.8}$$

where $k \sim M_{pl}$ is the gravity scale, and $b_0^{-1} \sim k/24\pi$ is the RS compactification scale. The exponential factor, $e^{-2kb_0|y|}$, in Eq.(2.8) is called the warp factor. One of the most important results of the RS model is the warp factor, $e^{-\frac{1}{2}kb_0}$, suppression of the KK mode's mass for the bulk fields.

Following the procedure developed in Ref. [5,23], we derive the mode expansion of the bulk gauge fields in RS space-time such that

$$A_\mu(x, y) = \sum_n A_\mu^{(n)}(x) \hat{\chi}_n(y), \tag{2.9}$$

and the mode functions are

$$\hat{\chi}_n(y) = \frac{\sqrt{kb_0} e^{kb_0|y|}}{N_n} \left[J_1 \left(\frac{M_n}{k} e^{kb_0|y|} \right) - \frac{J_0(M_n/k)}{Y_0(M_n/k)} Y_1 \left(\frac{M_n}{k} e^{kb_0|y|} \right) \right]. \tag{2.10}$$

where the KK mode's mass M_n is given by the solution of

$$J_0 \left(\frac{M_n}{k} \right) Y_0 \left(\frac{M_n}{k} e^{\frac{1}{2}kb_0} \right) = Y_0 \left(\frac{M_n}{k} \right) J_0 \left(\frac{M_n}{k} e^{\frac{1}{2}kb_0} \right), \tag{2.11}$$

and N_n is a normalization factor:

$$N_n^2 = z^2 \left[J_1 \left(\frac{M_n}{k} z \right) - \frac{J_0(M_n/k)}{Y_0(M_n/k)} Y_1 \left(\frac{M_n}{k} z \right) \right]^2 \bigg|_{Z=1}^{Z=\exp(kb_0/2)}. \tag{2.12}$$

For $kb_0 \gg 1$ and $M_n \ll k$ the KK mode's mass behaves asymptotically like [26]

$$M_n \sim \left(n - \frac{1}{4}\right) \pi k e^{-\frac{1}{2} k b_0}. \quad (2.13)$$

In this case, n dependence of the mode function $\hat{\chi}_n(y)$ is negligible in the brane $y = 1/2$. It behaves as [27]

$$\hat{\chi}_n(1/2) \sim \sqrt{k b_0}. \quad (2.14)$$

Performing the KK modes decomposition (2.9), the induced four-dimensional Lagrangian for a free gauge field reads

$$\begin{aligned} \mathcal{L}_{\text{gauge}}^{(4)} &= \int_0^{\frac{1}{2}} dy \left[-\frac{1}{4} \sqrt{-G} F^{MN} F_{MN} + \mathcal{L}_{\text{G.F.}} \right] \\ &= \frac{1}{2} \sum_n A_\mu^{(n)} \left[\eta^{\mu\nu} (\partial^2 + M_n^2) - (1 - \xi) \partial^\mu \partial^\nu \right] A_\nu^{(n)}. \end{aligned} \quad (2.15)$$

As is mentioned in the previous subsection, the fifth component of the gauge fields disappears by the Z_2 symmetry and the gauge fixing condition.

Thus, we find the four-dimensional Lagrangian with massless fermion,

$$\begin{aligned} \mathcal{L}^{(4)} &= \frac{1}{2} \sum_n A_\mu^{(n)} \left[\eta^{\mu\nu} (\partial^2 + M_n^2) - (1 - \xi) \partial^\mu \partial^\nu \right] A_\nu^{(n)} \\ &\quad + \bar{\psi} \left(i \partial_\mu + e A_\mu^{(0)} \right) \gamma^\mu \psi + e \sum_{n \neq 0} \hat{\chi}_n(y^*) \bar{\psi} A_\mu^{(n)} \gamma^\mu \psi. \end{aligned} \quad (2.16)$$

In the warped extra dimension the contribution from the extra dimension appears in the coupling constant of the interactions between the KK modes and the fermion. From Eq. (2.14) the KK modes have strong coupling in the brane for large $k b_0 (\gg 1)$ [5].

In the following section we study the structure of chiral symmetry breaking starting from the Lagrangian (2.1), (2.7) and (2.16).

III. LOCAL FOUR-FERMI APPROXIMATION

As a first step to investigate the phase structure, we assume that the KK mode's mass M_n is on the order of the cut-off scale Λ analytically to deal with chiral symmetry breaking. If all the KK excited modes are heavier enough, terms with KK modes in the Lagrangian (2.7) and (2.16) reduce to four-fermion interactions. After integrating out the KK excited modes, we obtain

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4} F^{(0)\mu\nu} F_{\mu\nu}^{(0)} + \bar{\psi} \left(i \partial_\mu + e A_\mu^{(0)} \right) \gamma^\mu \psi - \frac{1}{2} G \left(\bar{\psi} \gamma^\mu \psi \right) \left(\bar{\psi} \gamma_\mu \psi \right). \quad (3.1)$$

The four-fermion coupling constant G is given by

$$G = \sum_{n \neq 0}^{N_{KK}} \frac{4\pi\alpha R^2}{n^2} \xrightarrow{N_{KK} \rightarrow \infty} \frac{2\pi^2\alpha R^2}{3}, \quad (\text{for flat extra dimension}) \quad (3.2)$$

$$G = \sum_{n \neq 0}^{N_{KK}} \frac{4\pi\alpha}{M_n^2} \hat{\chi}_n(y^*)^2. \quad (\text{for warped extra dimension}) \quad (3.3)$$

If the gauge field is confined in the brane, the four-fermion interaction term is not generated, i.e. $G = 0$.

Using the Fierz transformation, the four-fermion interaction terms in (3.1) are rewritten as

$$\mathcal{L}_{\text{eff}} = -\frac{1}{4}F^{(0)\mu\nu}F_{\mu\nu}^{(0)} + \bar{\psi} \left(i\partial_\mu + eA_\mu^{(0)} \right) \gamma^\mu \psi + \frac{1}{2}G \left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 \right], \quad (3.4)$$

where we ignore the tensor type interactions. Eq. (3.4) is nothing but the gauged Nambu-Jona-Lasinio (NJL) model. A variety of methods have been developed to evaluate the phase structure in the gauged NJL model, (see [30,31]). According to the bifurcation method, the critical line in the α - G plane is analytically obtained [28,29]

$$\frac{G_c\Lambda^2}{4\pi^2} = \frac{1}{4} \left(1 + \sqrt{1 - \frac{3\alpha_c}{\pi}} \right)^2, \quad \left(\alpha < \frac{\pi}{3} \right) \quad (3.5)$$

where G_c and α_c denote the critical values of G and α respectively. For $\alpha < \pi/3$ and $G > G_c$, the fermion mass function $B(p^2)$ behaves as

$$B(p^2) = \frac{\sqrt{1-\omega^2}}{\omega} \frac{\mu^2}{\sqrt{p^2}} \sinh \left(\frac{\omega}{2} \ln \frac{p^2}{\mu^2} + \tanh^{-1}\omega \right), \quad (3.6)$$

where $\omega = \sqrt{1 - 3\alpha/\pi}$ and μ is given by

$$\mu = \Lambda \exp \left[-\frac{1}{\omega} \tanh^{-1} \left(\frac{\omega/2}{-1/4 - \omega^2/4 + G\Lambda^2/(4\pi^2)} \right) \right]. \quad (3.7)$$

The coupling constants α and G are not independent parameters for brane world models considered here. Substituting Eqs. (3.2) and (3.3) into Eq.(3.5), we find the critical coupling constant α_c . In Tab. I we summarize values of the critical coupling constant for some characteristic cases. Since the local four-fermi approximation must be valid for heavy KK excitations, we consider the case $R \sim 1/\Lambda$ for the flat extra dimension and $kb_0 \sim \pi$ for the warped extra dimension where only the lightest KK mode has mass below the cut-off scale. In the flat extra dimension the lightest KK excitation mode is almost decoupled. Thus, the KK modes have only a small correction to enhance chiral symmetry breaking. The result is similar to that without KK modes, i.e. QED. On the other hand, the critical coupling in warped extra dimension is less than a half value of that in QED in the $y = 1/2$ brane, because the coupling constants between the fermion and the KK modes are enhanced by the mode expansion function $\hat{\chi}(y^*)$.

In Fig. 1 we show the behavior of the mass function $B(p^2)$ as a function of the coupling constant α for a fixed p . The mass function smoothly disappears as the coupling constant α decreases. In other words, no mass gap is observed. Thus, the second order phase transition is realized. Such a behavior is known as a generalization of the Miransky scaling. On the $y = 0$ brane in the warped extra dimension the effective coupling constant of the lightest KK mode is almost α , and the situation is similar to the brane in the flat extra dimension. Therefore, in the following calculations, we focus on the $y = 1/2$ brane for the warped extra dimension and the $y = 0$ brane is ignored.

IV. NUMERICAL ANALYSIS OF THE SCHWINGER-DYSON EQUATION

In the previous section we assumed that the KK excited modes were heavy enough, and we studied the gauged NJL model as a low-energy effective theory on the brane. The four-fermi approximation may be valid for heavy KK modes. To examine the validity of the approximation in the previous section, we introduce the Schwinger-Dyson (SD) equation [32].

$$\begin{aligned} iS^{-1}(p) &\equiv A(-p^2)\not{p} - B(-p^2) \\ &= \not{p} + \sum_{n=0}^{N_{KK}} \int \frac{d^4q}{i(2\pi)^4} e^2 \gamma^\mu S(q) D_{\mu\nu}^{(n)}(q-p) \Gamma_\nu^{(n)}(q, p; q-p), \end{aligned} \quad (4.1)$$

where $S(q)$, $D_{\mu\nu}^{(n)}(q-p)$ represents the full propagators for the fermion and the n-th KK excitation modes, and $\Gamma_\nu^{(n)}(q, p; q-p)$ is the full vertex function. To solve this equation, we adopt the ladder approximation in which the propagators, $D_{\mu\nu}^{(n)}(q-p)$, and the vertex parts, $\Gamma_\nu^{(n)}(q, p; q-p)$, are approximated by tree forms. Within the ladder approximation, the SD equation reduces to the self-consistent equation for the full fermion propagators, $S(p)$. After some calculations the ladder SD equation reads [33]

$$A(p^2) = 1 + \int_0^{\Lambda^2} dq^2 \frac{q^2 A(q^2)}{A^2(q^2)q^2 + B^2(q^2)} \sum_{n=0}^{N_{KK}} L_\xi(p^2, q^2; M_n, \alpha_n), \quad (4.2)$$

$$B(p^2) = \int_0^{\Lambda^2} dq^2 \frac{q^2 B(q^2)}{A^2(q^2)q^2 + B^2(q^2)} \sum_{n=0}^{N_{KK}} K_\xi(p^2, q^2; M_n, \alpha_n), \quad (4.3)$$

where ξ is a gauge-fixing parameter (see appendix) and

$$\begin{aligned} L_\xi(p^2, q^2; M_n, \alpha_n) &= \frac{\alpha_n}{4\pi} \left[q^2 f_{M_n^2}^2(p^2, q^2) + \frac{2q^2}{M_n^2} \{ f_{M_n^2}(p^2, q^2) - f_{\xi M_n^2}(p^2, q^2) \} \right. \\ &\quad \left. + \frac{(q^2)^2 + p^2 q^2}{2M_n^2} \{ f_{M_n^2}^2(p^2, q^2) - f_{\xi M_n^2}^2(p^2, q^2) \} \right], \end{aligned} \quad (4.4)$$

$$\begin{aligned} K_\xi(p^2, q^2; M_n, \alpha_n) &= \frac{\alpha_n}{4\pi} \left[4f_{M_n^2}(p^2, q^2) + \frac{p^2 + q^2}{M_n^2} \{ f_{M_n^2}(p^2, q^2) - f_{\xi M_n^2}(p^2, q^2) \} \right. \\ &\quad \left. - \frac{p^2 q^2}{2M_n^2} \{ f_{M_n^2}^2(p^2, q^2) - f_{\xi M_n^2}^2(p^2, q^2) \} \right], \end{aligned} \quad (4.5)$$

and

$$f_M(p^2, q^2) = \frac{2}{p^2 + q^2 + M + \sqrt{(p^2 + q^2 + M)^2 - 4p^2 q^2}}. \quad (4.6)$$

The modified coupling constant α_n is equal to α in the flat extra dimension. In the warped extra dimension α_n is given by

$$\alpha_n \equiv \alpha \hat{\chi}_n(y^*)^2, \quad \alpha_0 = \alpha. \quad (4.7)$$

The function $B(p^2)$ is the traceless part of the fermion self-energy. It corresponds to the fermion mass function. If $B(p^2)$ develops a non-vanishing vacuum expectation value, the fermion acquires the dynamical mass, and the chiral symmetry is broken down dynamically. Thus we can regard the mass function, $B(p^2)$, as the order parameter of chiral symmetry breaking. To study if the fermion acquires a dynamical mass or not, we numerically analyze the simultaneous integral equations (4.2) and (4.3) in terms of bare quantities.

First, we analyze the flat extra dimension with $R \sim 1/\Lambda$ and the $y = 1/2$ brane in the warped extra dimension with $kb_0 \sim \pi$. Since only the lightest KK mode mass is less than the cut-off scale, it is natural to take $N_{KK} = 1$. To solve the Eqs. (4.2) and (4.3) numerically, we employ the iteration method. Starting from suitable trial functions for $A(p^2)$ and $B(p^2)$, we numerically evaluate the right-hand sides of Eqs. (4.2) and (4.3). We next use the resulting functions, $A(p^2)$ and $B(p^2)$, to calculate the right-hand sides of Eqs. (4.2) and (4.3), and we iterate the calculational procedure until stable solutions are obtained. At each iteration, the integration is performed by using the Monte Carlo method, and it is cut off at mass scale Λ .

If the gauge fields do not propagate in the extra dimension, i.e. no KK modes exist, the Landau gauge, $\xi = 0$, is consistent with the Ward-Takahashi identity for the ladder SD equation. In the bulk gauge theories massive KK modes appear in the brane, and the Ward-Takahashi identity is not guaranteed within the ladder approximation in the Landau gauge. We try to solve the ladder SD equation by varying the gauge parameter ξ . As is illustrated in Fig. 2, it is possible to choose the gauge parameter $\xi = \xi_{WT}$ for which the fermion wave function $A(p^2)$ is almost unity, and the Ward-Takahashi identity nearly holds on the brane. In the models considered here, ξ_{WT} is given by

$$\xi_{WT} \sim 0.10, \quad (\text{for flat extra dimension, } R \sim 1/\Lambda) \quad (4.8)$$

$$\xi_{WT} \sim 0.31. \quad (\text{for warped extra dimension, } kb_0 \sim \pi, y = 1/2) \quad (4.9)$$

Below we choose the gauge parameter $\xi = \xi_{WT}$. In Figs. 3 and 4, the typical behaviors of $B(p^2)$ normalized by the cutoff scale Λ are presented as a function of the momentum p for fixed α .

To study the structure of the chiral phase transition, we observe the behavior of the mass function $B(p^2)$ at some fixed value of p . The α dependence of the mass function $B(p)$ with $p = 0.01\Lambda$ is drawn in Fig. 5. As is seen in Fig. 5, the chiral phase transition is of the second order (or continuous), since a fermion mass is generated at a critical value of the coupling constant α without any discontinuity. For the flat extra dimension, $M^4 \otimes S^1/Z_2$, the lightest KK mode increases the mass function $B(p)$, but the result has no large difference from QED. The result for the warped extra dimension is far from the other cases. In the $y = 1/2$ brane the critical coupling α_c is almost half of that in QED. It comes from the large effective coupling $\alpha_1 \sim 4\alpha$ for the lightest KK mode. Comparing Fig. 1 with Fig. 5, we observe that the result of the ladder SD equation is qualitatively consistent with that obtained by the local four-fermi approximation in the previous section.

Next, we study how the number of KK modes, N_{KK} , affects the chiral phase transition. N_{KK} is defined as the number of KK modes whose mass is less than the cut-off scale Λ . Here we consider the SD equation in the flat extra dimension with $R \sim 5/\Lambda, 10/\Lambda, 15/\Lambda, 20/\Lambda$. Obviously N_{KK} for each radii is given by 5, 10, 15 and 20 respectively. In the warped extra dimension KK mode's mass depends on the RS compactification scale, b_0 , which is defined in Eq. (2.8). We also evaluate the SD equation in the warped extra dimensions where N_{KK}

corresponds to 5, 10, 15 and 20 each. By solving Eq. (2.11) it is found that these N_{KK} are realized for $kb_0 \sim 1.80\pi, 2.22\pi, 2.47\pi$ and 2.65π respectively. Here we assume that the heaviest KK mode's mass is near the cut-off scale.

In models with $N_{KK} = 5, 10, 15$ and 20 , the gauge parameter for which the fermion wave function is almost unity is give by $\xi_{WT} \sim 0.27$. Taking the gauge parameter $\xi = \xi_{WT}$, we solve the Eqs. (4.2) and (4.3) numerically. In Figs. 6 and 7 we illustrate the behaviors of the mass function $B(p^2)$ for the flat extra dimension and for the $y = 1/2$ brane in the warped extra dimension. In both brane models, chiral symmetry breaking is enhanced by KK mode summation. For the $y = 1/2$ brane in the warped extra dimension the critical coupling approaches the ordinary electromagnetic coupling constant as N_{KK} increases.

It is expected that chiral symmetry breaking is extremely enhanced by the warp factor for larger kb_0 . There is a possibility that the chiral symmetry is broken down in ordinary electromagnetic coupling constant in the warped extra dimension for large N_{KK} .

V. CONCLUSION

We have investigated the mechanism of chiral symmetry breaking in Abelian gauge theories with massless fermion in the brane. We consider brane worlds in the space-times $M^4 \otimes S^1/Z_2$ and AdS^5 (RS model). The induced four-dimensional Lagrangian are constructed from the five-dimensional theory in the brane world. We adopt the gauge fixing condition to decouple the extra component of the gauge field. In this case the contribution of the extra dimension appears as massive KK excitation modes for ordinary gauge fields in the brane. It is very important for chiral symmetry breaking that the coupling constant between KK modes and fermions becomes strong in the $y = 1/2$ brane in the RS model.

To study the influence of the KK modes on the chiral phase transition, we assume that the KK modes are heavy enough, and consider the gauged NJL model as a low-energy effective theory. According to the bifurcation method, chiral symmetry breaking is of the second order. In the brane included in a flat extra dimension and the $y = 0$ brane in a warped extra dimension, heavy KK modes are almost decoupled. They have only a small effect on chiral symmetry breaking. However, it is found that chiral symmetry breaking is extremely enhanced in the $y = 1/2$ brane in the warped extra dimension due to the large effective coupling. For example, the critical coupling in the warped extra dimension with $kb_0 = \pi$ is almost half of that in QED in the $y = 1/2$ brane.

In the gauged NJL model, propagators of the KK modes are approximated by the local four-fermion interactions. To justify this approximation we consider the SD equation including the effect of the lightest KK mode propagation. We choose the gauge parameter to hold the Ward-Takahashi identity, and we numerically solve the ladder SD equation. The result is qualitatively consistent with that in the gauged NJL model for $N_{KK} = 1$. For the gauged NJL model we pinch the KK modes propagation, and use the bifurcation method where the infrared part of the integral is cut off at the mass scale μ . Thus, the dynamical fermion mass and the critical coupling constant is not explicitly equivalent in both the calculations. The effective couplings of the KK modes become stronger and the KK mode's mass becomes lighter as kb_0 increases in the $y = 1/2$ brane in the warped extra dimension.

The KK modes summation enhances the chiral symmetry breaking. The critical value of the coupling constant decreases as N_{KK} increases. If the chiral symmetry is broken

down for ordinary electromagnetic coupling constant, all the charged fermions may acquire a dynamical mass. But it takes long time to solve the ladder SD equation numerically as N_{KK} increases. Furthermore, to estimate the scale of the dynamical fermion mass, we must solve the SD equation in terms of renormalized quantities. This will be the subject of a forthcoming paper.

The other improvement we should make is to extend our analysis to a variety of brane world scenarios, and to construct a model of dynamical symmetry breaking in grand unified theories. It is also interesting to apply our results to the critical phenomena of electroweak symmetry breaking. We will continue our work and we hope to report on these problems.

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APPENDIX A: GAUGE FIXING CONDITION

In this appendix we present the gauge fixing terms for the bulk Abelian gauge field with the brane. We apply the procedure which is known as R_ξ gauge in the massive gauge theory with the spontaneously broken gauge symmetry. First, we consider the common covariant gauge fixing term in $(4 + d)$ -dimensional space-time, with the Lorentz indices $M, N = 0, 1, 2, \dots, 3 + d$. The term is written as

$$\begin{aligned}\mathcal{L}_{\text{GF+FP}} &= -i\delta_B \left[\bar{c} \left(\partial^M A_M + \frac{1}{2}\alpha B \right) \right] \\ &= B \partial^M A_M + \frac{1}{2}\alpha B^2 + i\bar{c} \partial^M \partial_M c \\ &= \frac{1}{2}\alpha \left(B + \frac{1}{2}\partial^M A_M \right)^2 - \frac{1}{2\alpha} \left(\partial^M A_M \right)^2 + i\bar{c} \partial^M \partial_M c,\end{aligned}\tag{A1}$$

where δ_B represents the BRST transformation, B is the Nakanishi-Lautrup field and α is the gauge fixing parameter. This term is covariant under the full $(4 + d)$ -dimensional Lorentz transformation.

Next we want to consider the case in which there exist 3-branes in the direction of the Lorentz index $\mu = 0, 1, 2, 3$. The 3-brane breaks the Lorentz symmetry. In this case the gauge fixing term can be written in the form

$$\begin{aligned}\mathcal{L}_{\text{GF+FP}} &= -i\delta_B \left[\bar{c} \left(\partial^\mu A_\mu + \beta \partial^m A_m + \frac{1}{2}\alpha B \right) \right] \\ &= B (\partial^\mu A_\mu + \beta \partial^m A_m) + \frac{1}{2}\alpha B^2 + i\bar{c} \partial^\mu \partial_\mu c + i\beta \bar{c} \partial^m \partial_m c\end{aligned}$$

$$\begin{aligned}
&= \frac{1}{2}\alpha \left(B + \frac{1}{\alpha}\partial^\mu A_\mu + \frac{\beta}{\alpha}\partial^m A_m \right)^2 - \frac{1}{2\alpha} (\partial^\mu A_\mu + \beta\partial^m A_m)^2 \\
&\quad + i\bar{c}\partial^\mu\partial_\mu c + i\beta\bar{c}\partial^m\partial_m c,
\end{aligned} \tag{A2}$$

where β is another gauge fixing parameter, $\mu, \nu = 0, 1, 2, 3$ and $m, n = 4, 5, \dots, 3 + d$. The second term in the last line of Eq. (A2) is similar to the usual R_ξ gauge which cancels out the mixing with Nambu-Goldstone boson. After eliminating the Nakanishi-Lautrup field B , we obtain

$$\begin{aligned}
\mathcal{L}_{\text{GF+FP}} &= -\frac{1}{2\alpha} (\partial^\mu A_\mu + \beta\partial^m A_m)^2 + i\bar{c}\partial^\mu\partial_\mu c + i\beta\bar{c}\partial^m\partial_m c \\
&= -\frac{1}{2\alpha} (\partial^\mu A_\mu)^2 - \frac{\beta}{\alpha} (\partial^\mu A_\mu) (\partial^m A_m) - \frac{\beta^2}{2\alpha} (\partial^m A_m)^2 \\
&\quad + i\bar{c}\partial^\mu\partial_\mu c + i\beta\bar{c}\partial^m\partial_m c.
\end{aligned} \tag{A3}$$

On the other hand, the Lagrangian for the bulk Abelian gauge field contains the following terms:

$$\begin{aligned}
-\frac{1}{4}F^{MN}F_{MN} &= \frac{1}{2}A_M \left(\eta^{MN}\partial^2 - \partial^M\partial^N \right) A_N \\
&= \frac{1}{2}A_\mu \left(\eta^{\mu\nu}\partial^2 - \partial^\mu\partial^\nu \right) A_\nu - A_\mu\partial^\mu\partial^m A_m \\
&\quad + \frac{1}{2}A_m \left(\delta^{mn}\partial^2 - \partial^m\partial^n \right) A_n,
\end{aligned} \tag{A4}$$

where $\partial^2 = \partial^\mu\partial_\mu + \partial^m\partial_m$. If we choose $\beta = \alpha$, the second term in Eq. (A3) and that in Eq. (A4) are canceled out. The mixing terms between A_m and A_μ disappear. Thus, the extra components A_m and the Faddeev-Popov ghosts c and \bar{c} are decoupled from the brane electrodynamics.

Up to here we have treated the flat space-time case, however we can easily extend it to the curved background case, such as the Randall-Sundrum space-time, by replacing the derivatives with the general coordinate covariant derivatives. In this paper we use the gauge fixing terms discussed here with gauge fixing parameter $\xi = 1/\alpha$.

REFERENCES

- [1] I. Antoniadis, Phys. Lett. B **246**, 377 (1990).
- [2] I. Antoniadis and K. Benakli, Phys. Lett. B **326**, 69 (1994).
- [3] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. **B429**, 263 (1998); Phys. Rev. **D59**, 086004 (1999).
- [4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).
- [5] S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, Phys. Rev. **D62**, 084025 (2000).
- [6] Th. Kaluza, Sitzungsber. d. Preuss. Akad. d. Wiss., 966 (1921); O. Klein, Zeitsch. f. Phys. **37**, 895 (1926).
- [7] B. A. Dobrescu, Phys. Lett. **B461**, 99 (1999).
- [8] H. Cheng, B. A. Dobrescu and C. T. Hill, Nucl. Phys. B **589**, 249 (2000).
- [9] A. B. Kobakhidze, hep-ph/9904203.
- [10] H. Abe, H. Miguchi and T. Muta, Mod. Phys. Lett. **A15**, 445 (2000).
- [11] N. Rius and V. Sanz, hep-ph/0103086.
- [12] H. Abe, T. Inagaki and T. Muta, hep-ph/0104002.
- [13] N. Arkani-Hamed, H.-C. Cheng, B. A. Dobrescu and L. J. Hall, Phys. Rev. **D62**, 096006 (2000).
- [14] M. Hashimoto, M. Tanabashi and K. Yamawaki, hep-ph/0010260.
- [15] E. V. Gorbar, V. P. Gusynin and V. A. Miransky, hep-ph/0105059.
- [16] T. Inagaki, T. Muta and S. D. Odintsov, Prog. Theor. Phys. Suppl. **127**, 93 (1997).
- [17] K. Ishikawa, T. Inagaki, K. Fukazawa and K. Yamamoto, Prog. Theor. Phys. **99**, 237 (1998).
- [18] T. Inagaki, T. Muta and S. D. Odintsov, Mod. Phys. Lett. **A8**, 2117 (1993).
- [19] T. Inagaki, S. Mukaigawa and T. Muta, Phys. Rev. **D52**, 4267 (1995).
- [20] T. Inagaki, Int. J. of Mod. Phys. **A11**, 4561 (1996).
- [21] M. Baker, K. Johnson and R. S. Willey, Phys. Rev. Lett. **11**, 518 (1963); Phys. Rev. **136**, B1111 (1964); **163**, 1699 (1967).
- [22] T. Maskawa and H. Nakajima, Prog. Theor. Phys. **52**, 1326 (1974); **54**, 860 (1975).
- [23] W. D. Goldberger and M. B. Wise, Phys. Rev. Lett. **83**, 4922 (1999).
- [24] T. Kugo and K. Yoshioka, Nucl. Phys. B **594**, 301 (2001).
- [25] T. Han, J. D. Lykken and R.-J. Zhang, Phys. Rev. **D59**, 105006 (1999).
- [26] A. Pomarol, Phys. Lett. **85**, 4004 (2000).
- [27] J. Kubo, H. Terao and G. Zoupanos, Nucl. Phys. **574**, 495 (2000).
- [28] T. Appelquist, M. Soldate, T. Takeuchi and L. C. R. Wijewardhane, in *Proc. Johns Hopkins Workshop on Current Problems in Particle Theory 12, TeV Physics*, eds. G. Domokos and S. Kovesi-Domokos (World Scientific Pub. Co., 1988).
- [29] K. -I. Kondo, H. Mino and K. Yamawaki, Phys. Rev. **D39**, 2430 (1989).
- [30] K.-I. Kondo, in *Proc. 1991 Nagoya Spring School on Dynamical Symmetry Breaking*, ed. K. Yamawaki (World Scientific Pub. Co., 1992).
- [31] See, e.g. V. A. Miransky, *Dynamical Symmetry Breaking in Quantum Field Theories* (World Scientific Pub. Co., 1993) for a full list of references.
- [32] F. J. Dyson, Phys. Rev. **75**, 1736 (1949),
J. Schwinger, *Proc. Natl. Acad. Sci. USA* **37**, 452, 455 (1951).

- [33] M. Bando, in *Proc. 1991 Nagoya Spring School on Dynamical Symmetry Breaking*, ed. K. Yamawaki (World Scientific Pub. Co., 1992).

TABLES

	N_{KK}	Radius	M_1	α_1	α_c
QED	0			0	$\pi/3 \sim 1.05$
Flat extra dim.	1	$R\Lambda = 1$	$\sim \Lambda$	α	1.03
Flat extra dim.	∞	$R\Lambda = 1$	$\sim \Lambda$	α	1.01
Warped extra dim. ($y=1/2$)	1	$kb_0 = \pi$	$\sim 0.80k$	$\sim 4.0\alpha$	0.40
Warped extra dim. ($y=0$)	1	$kb_0 = \pi$	$\sim 0.80k$	$\sim 0.9\alpha$	0.94

TABLE I. The critical couplings α_c evaluated from the result of the four-fermi approximation for $N_{KK} = 1$. R is the radius of the flat extra dimension. The radius of warped extra dimension, r , is defined as $b_0 \equiv 2\pi R$, and $k = \Lambda$. In the third line, $N_{KK} \rightarrow \infty$ in the flat extra dimension, meaning that the summation of the KK mode is taken to be infinity but Λ is fixed to be finite. This corresponds to an anisotropy between the cut-off of the bulk and of the brane.

FIGURES

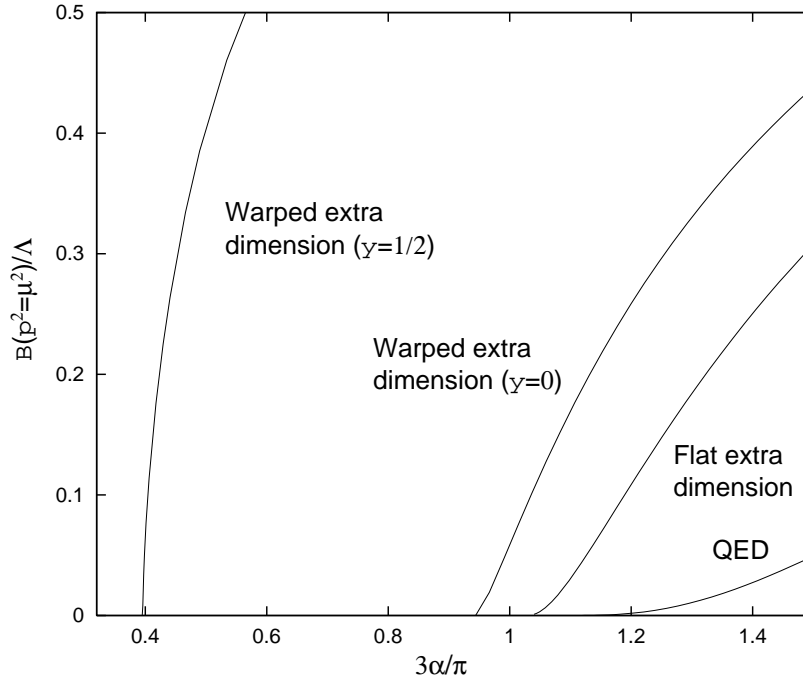


FIG. 1. Behavior of the mass function $B(p = \mu)$ as a function of α for QED, flat extra dimension ($R \sim 1/\Lambda$) and the warped extra dimension ($kb_0 \sim \pi$) in the gauged NJL model.

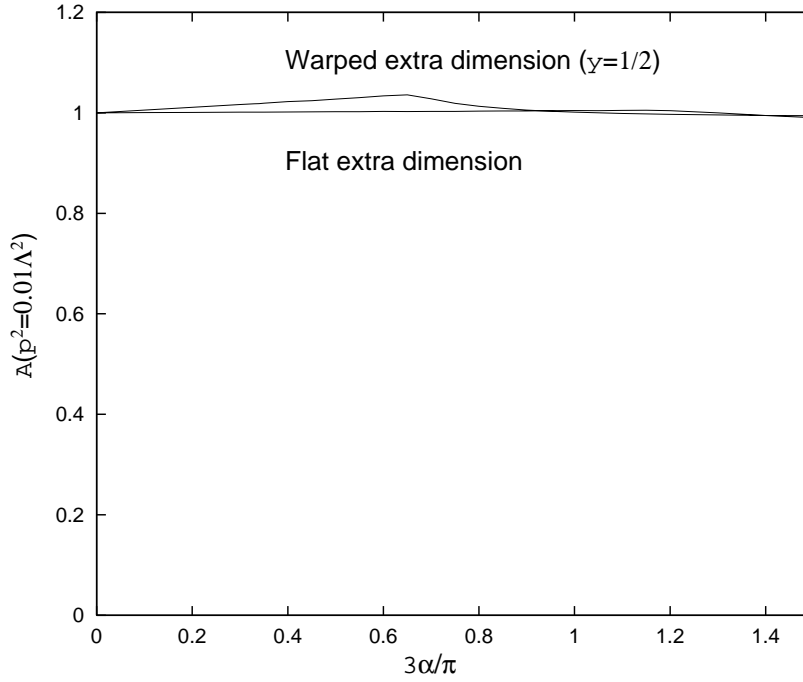


FIG. 2. Behavior of the wave function $A(p = 0.01\Lambda)$ as a function of the coupling constant α for flat extra dimension ($R \sim 1/\Lambda$) with $\xi \sim 0.10$ and the warped extra dimension ($kb_0 \sim \pi$) with $\xi \sim 0.31$.

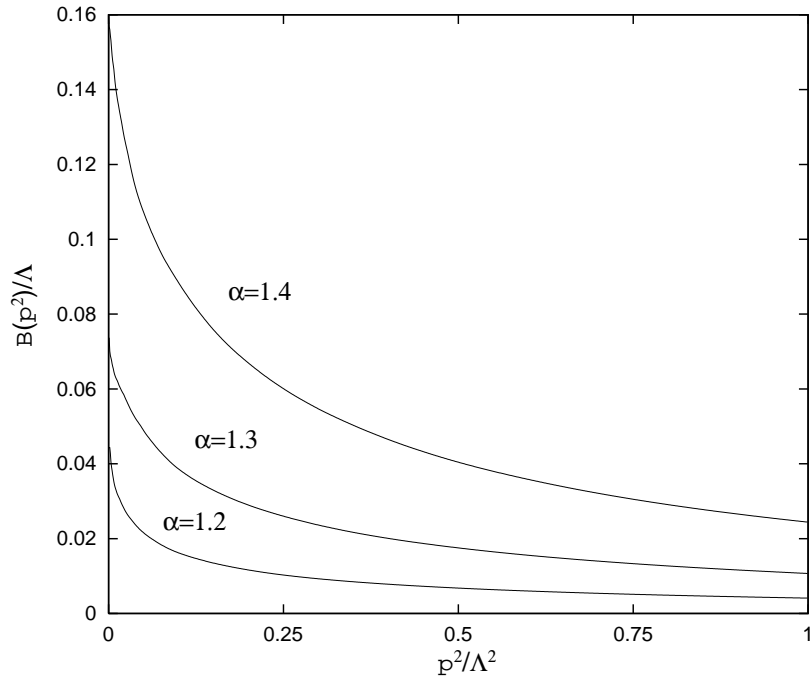


FIG. 3. Behavior of the mass function $B(p)$ as a function of the momentum p in the flat extra dimension with the coupling constant α fixed at 1.2, 1.3 and 1.4.

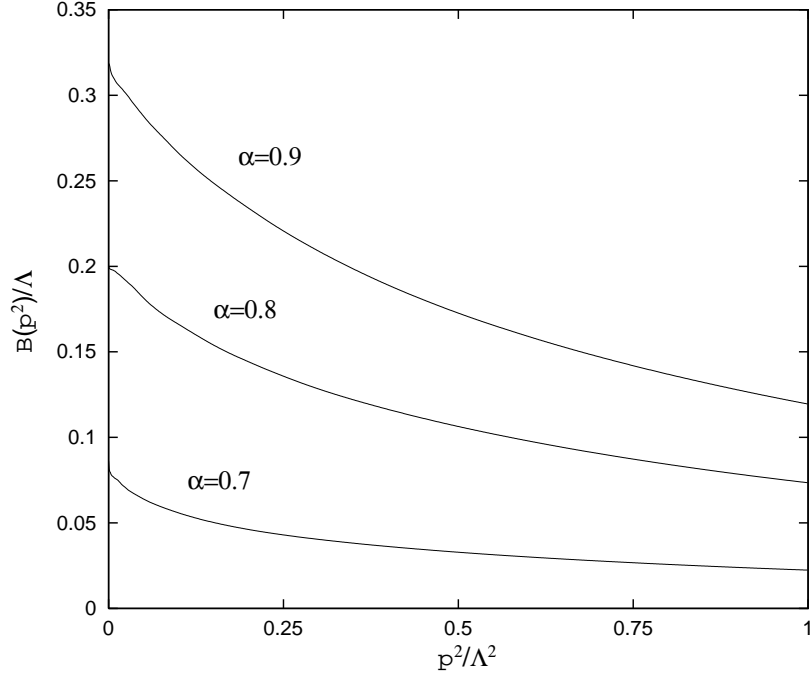


FIG. 4. Behavior of the mass function $B(p)$ as a function of the momentum p in the $y = 1/2$ brane in the warped extra dimension with the coupling constant α fixed at 0.7, 0.8 and 0.9.

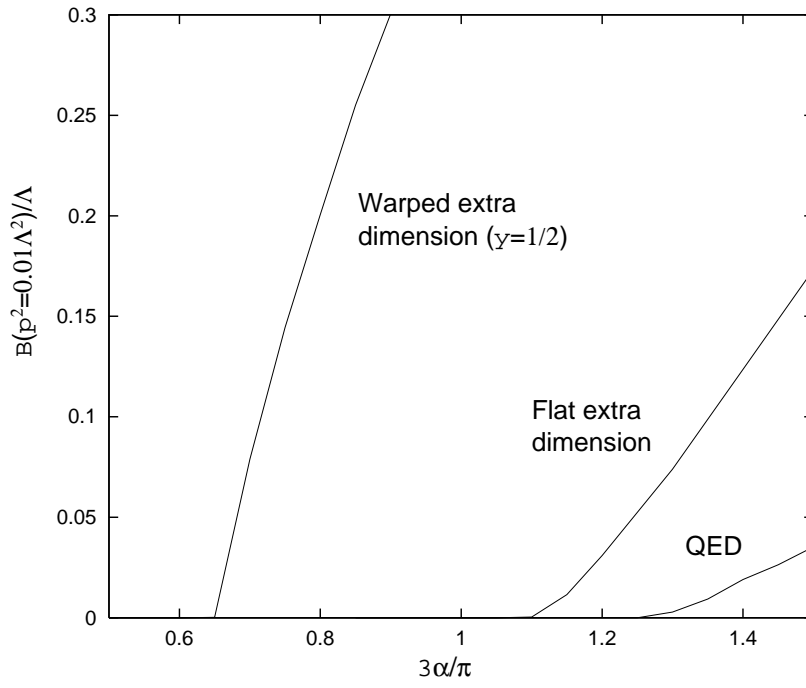


FIG. 5. Behavior of the mass function $B(p = 0.01\Lambda)$ as a function of α for QED, flat extra dimension ($R \sim 1/\Lambda$) and the warped extra dimension ($kb_0 \sim \pi$) in the ladder SD equation.

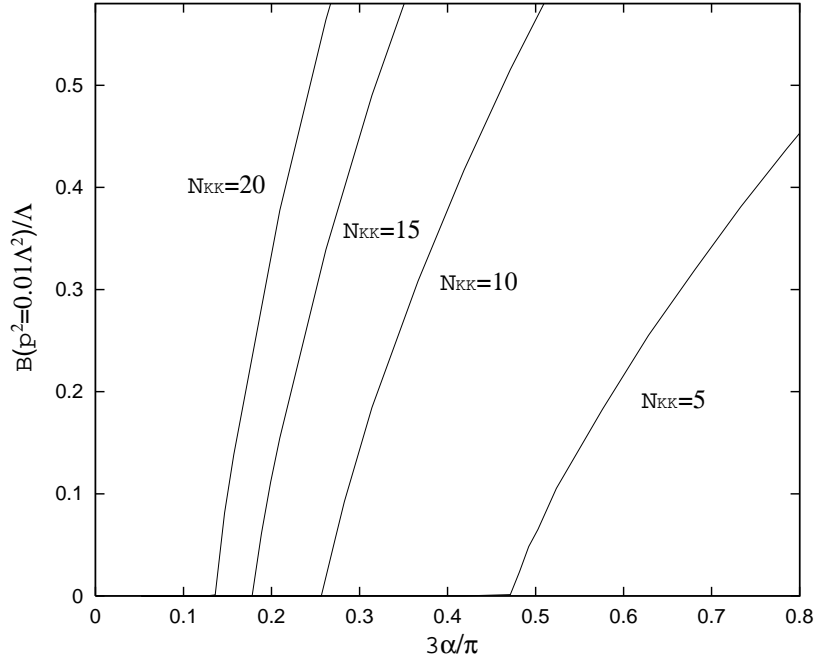


FIG. 6. Behavior of the mass function $B(p = 0.01\Lambda)$ as a function of α for flat extra dimension with $R = 5/\Lambda(N_{KK} = 5)$, $R = 10/\Lambda(N_{KK} = 10)$, $R = 15/\Lambda(N_{KK} = 15)$ and $R = 20/\Lambda(N_{KK} = 20)$ in the ladder SD equation.

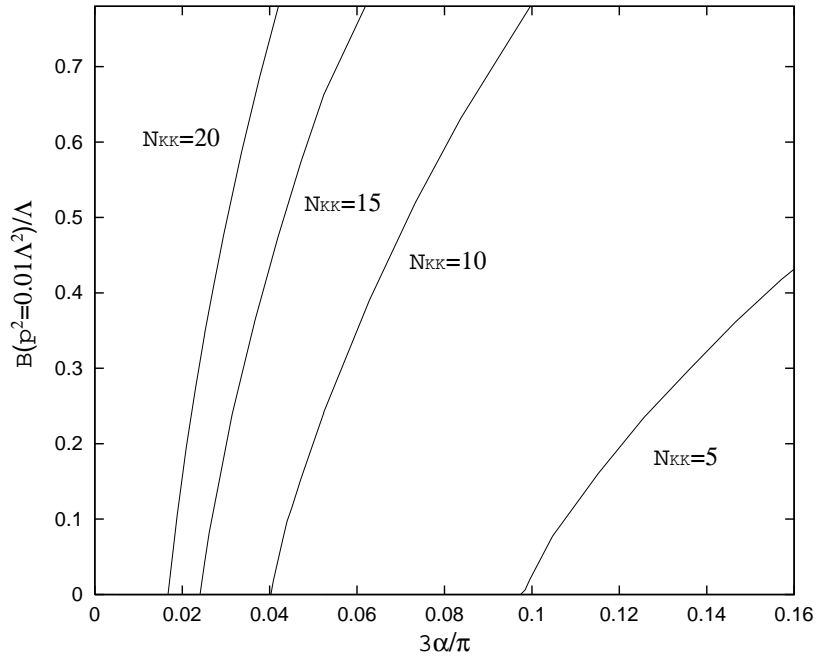


FIG. 7. Behavior of the mass function $B(p = 0.01\Lambda)$ as a function of α for the warped extra dimension ($y = 1/2$) with $kb_0 = 1.80\pi(N_{KK} = 5)$, $kb_0 = 2.22\pi(N_{KK} = 10)$, $kb_0 = 2.47\pi(N_{KK} = 15)$ and $kb_0 = 2.65\pi(N_{KK} = 20)$ in the ladder SD equation.